Lattice QCD and Quark Flavor Physics

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Why Lattice QCD?

The QCD coupling α_s runs with distance scale. At long distances, the theory is strongly coupled.

We need a reliable nonperturbative tool to calculate all the low energy phenomena of QCD, from the hadron spectrum to quark masses to weak matrix elements.

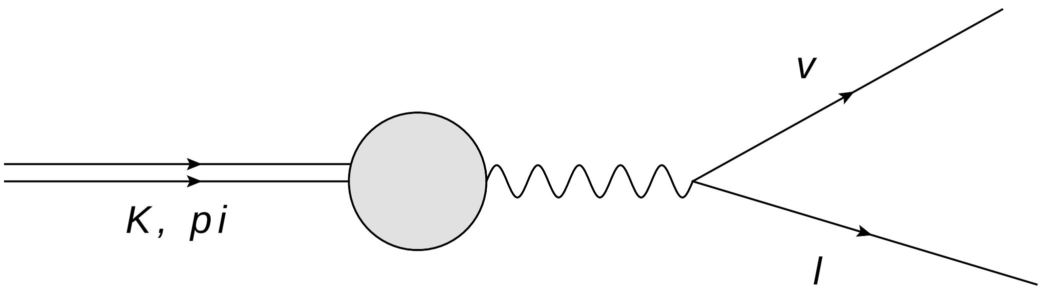
Precision determination of Standard Model hadronic parameters is required to make best use of experimental data from Belle, Babar, CLEO, BESIII, LHCb and others, as well as future facilities like Belle II and Project X. Most indirect physics searches require nonperturbative input.

Lattice QCD in the LHC era

If a “particle zoo” is discovered, ATLAS and CMS will measure the spectrum. Precision flavor measurements still important as part of studies to learn the underlying structure of the theory.

If new physics is beyond the reach of direct production at LHC, indirect searches using high precision low energy quantities may be our best bet to discover new physics.

Nonperturbative input needed



$$\Gamma = (\text{known factor})\,(\text{CKM factor})(\text{QCD factor}) \tag{1}$$

$$\frac{\Gamma(K \to \ell\overline{\nu}_\ell)}{\Gamma(\pi \to \ell\overline{\nu}_\ell)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K \left(1 - \frac{m_\ell^2}{m_K^2}\right)^2}{m_\pi \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2} \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right] \tag{2}$$

Lattice QCD Calculations

Calculate expectation values on an ensemble of gauge fields $[\mathcal{U}]$ with an exponential weight

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int D\mathcal{U} D\psi_{\rm sea} D\overline{\psi}_{\rm sea} e^{-S_{\rm QCD}[\mathcal{U},\psi_{\rm sea},\overline{\psi}_{\rm sea}]}\mathcal{O}[\mathcal{U},\psi_{\rm val},\overline{\psi}_{\rm val}], \qquad (3)$$

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int D\mathcal{U} \prod_{f=1}^{n_f} \mathrm{det}(\not{D} + m_f) e^{-S_{\rm QCD}[\mathcal{U}]}\mathcal{O}[\mathcal{U},\psi_{\rm val},\overline{\psi}_{\rm val}], \qquad (4)$$

The action is discretized, so that derivatives become finite differences. Integral is still too large to do directly ($N_s^3 \times N_t \times 4 \times N_f \times N_c$), so we use Monte Carlo importance sampling.

Computing

Cluster at Fermilab and BlueGene/Q at Argonne

Types of Errors

Because QCD with physical quark masses is a nonlinear multiscale problem ($\Lambda_{QCD} \approx 100 - 200$ MeV, $m_{u,d} \approx 2 - 6$ MeV, $m_b \approx 4.3$ GeV), it is very expensive to simulate at the physical quark masses.

1.) Statistics and fitting

2.) Tuning lattice spacing, a, and quark masses

3.) Matching lattice gauge theory to continuum QCD

4.) Extrapolation to continuum

5.) Chiral extrapolation to physical up, down quark masses

6.) Quenching. Uncontrolled!

Quenched Approximation

Configurations are generated with a weighting given by the gauge field and fermion determinant. Including the fermion determinant in this weighting is the most computationally demanding step in lattice QCD.

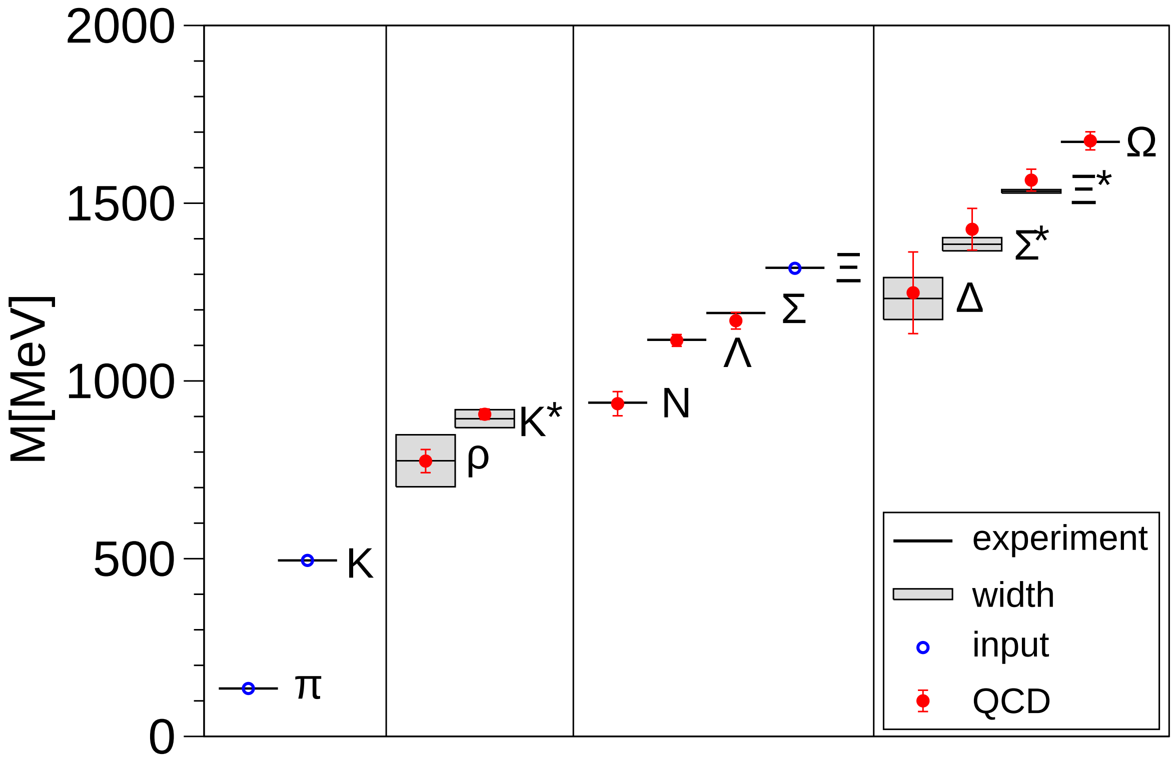
The quenched approximation ignores fermion-antifermion vacuum bubbles. This is an uncontrolled systematic error.

“Unquenched” calculations, where the fermion determinant is included, are now the norm.

Quenching the strange quark

- Strange threshold lies in the nonperturbative regime.
- Most quantities show no difference with 2+1 flavor results, with precision at the 3-5% level.
- Error is difficult to estimate. Could be as much as 5%, and only sure way to quantify it is to compare with 2+1 flavor results.
- Since the lattice world averages are approaching the level where quenching the strange is likely to be important, 2 flavor results are not usually included in averages.

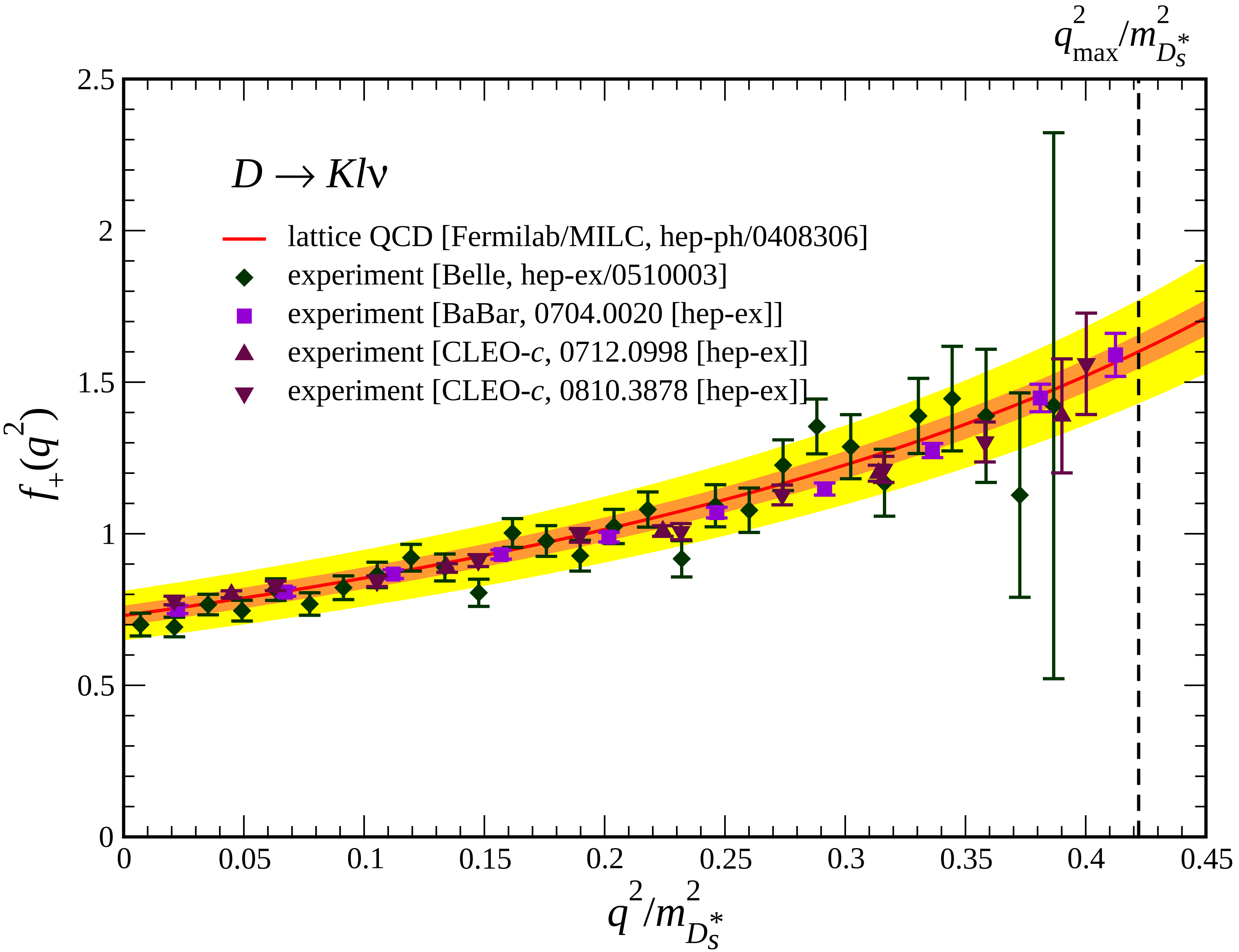
Hadron Spectrum from Lattice QCD



BMW Collaboration, Science 322:1224-1227, 2008.

- *Good agreement between theory and experiment!*

Prediction of form factor



Some types of lattice fermions

Lots of ways to solve the lattice fermion doubling problem:

- Wilson Fermions: Introduces an additional “irrelevant” term to the action. Improved variants, i.e. “clover” used in practice. Fairly cheap.
- Staggered Fermions: Identifies some of the extra fermions with the different spin components of a single fermion. There are still 4 extra species of fermions, and these are eliminated by taking the 4th root of the determinant. Some open theoretical issues with this, though theoretical progress has been made on this front. Very cheap.
- Domain Wall Fermions: Solves chiral symmetry problem by using Wilson type quarks in five dimensions. More costly because of the extra dimension. There is a small chiral symmetry breaking due to the finiteness of the fifth dimension. Expensive.
- Overlap Fermions: Exact lattice chiral symmetry. Very expensive.

Lattice Averages

Many lattice quantities have reached the mature stage of having controlled systematic errors and results from several groups using different methods.

www.latticeaverages.org for relatively recent updates of lattice averages based on JL, E Lunghi, R S Van de Water, Phys Rev D 81 034503 (2010) [arXiv:0910.2928]. FLAG(Flavor Lattice Averaging Group) also produced averages [EPJ C71:1695 (2011)]. Methodology differs somewhat, results are broadly consistent.

The two groups have merged into FLAG II.

Includes light and heavy quark physics quantities, many weak-matrix elements for flavor physics.

FLAG II members

- Advisory Board: S. Aoki, C. Bernard, C. Sachrajda
- Editorial Board: G. Colangelo, H. Leutwyler, T. Vladikas, U. Wenger
- Working Groups:
- Quark masses: L. Lellouch, T. Blum, V. Lubicz
- V_{us}, V_{ud}: A. Jüttner, T. Kaneko, S. Simula
- LEC's: S. Dürr, H. Fukaya, S. Necco
- B_K: H. Wittig, JL, S. Sharpe
- α_s: R. Sommer, T. Onogi, R. Horsley
- f_B, B_B: A. El Khadra, Y. Aoki, M. Della Morte
- B and D semileptonic form factors: R. Van de Water, E. Lunghi, C. Pena, J. Shigemitsu

What the lattice can do

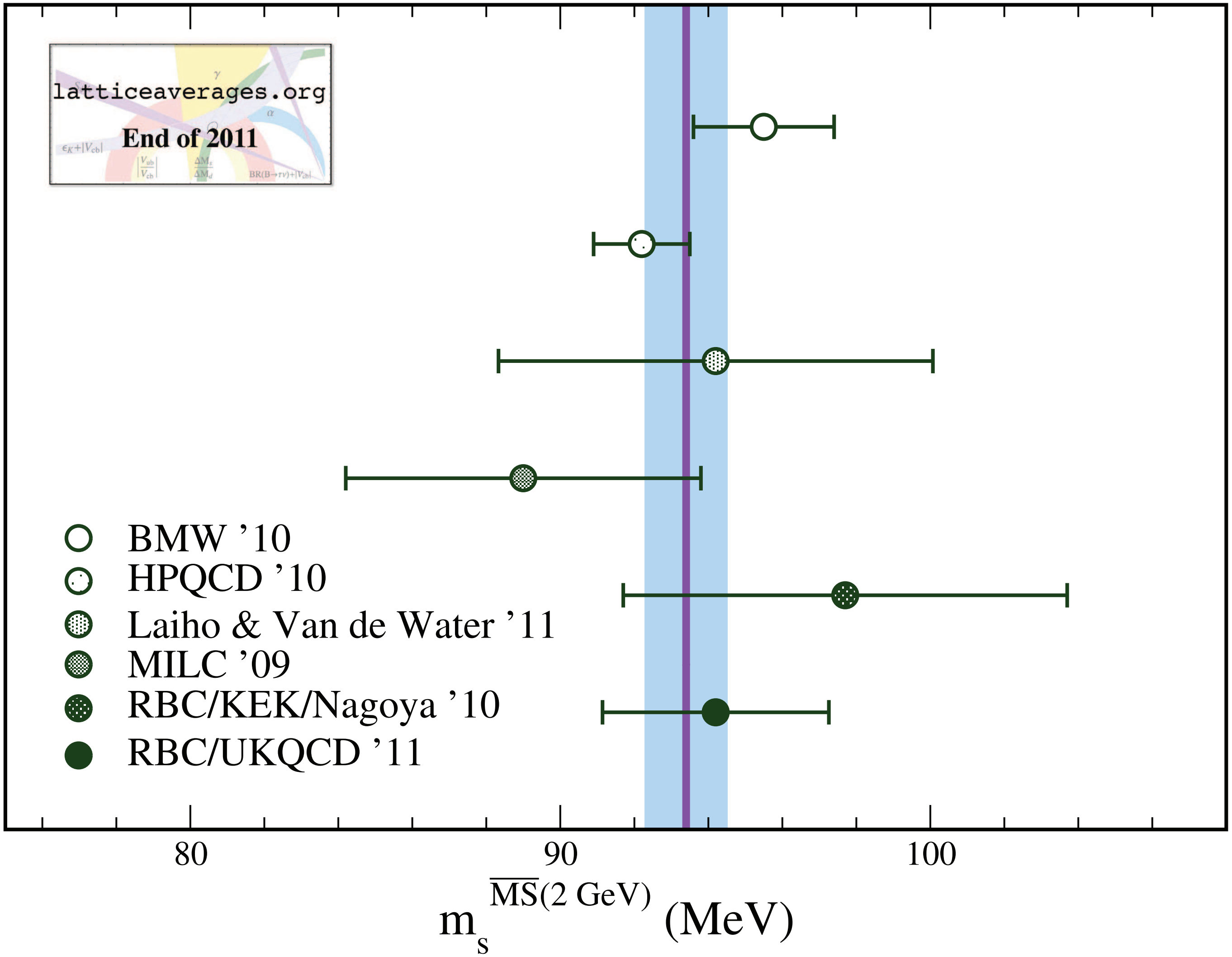
Lattice calculations are definitely doable when there is at most one hadron (stable under QCD) in the initial and final states. Baryons are more challenging than mesons. Unstable particles are very challenging.

Around twenty weak matrix elements have mature calculations, with existing results and expected improvements: f_K, f_π, $K \to \pi\ell\nu$, $D \to K\ell\nu$, $B \to \pi\ell\nu$, ...

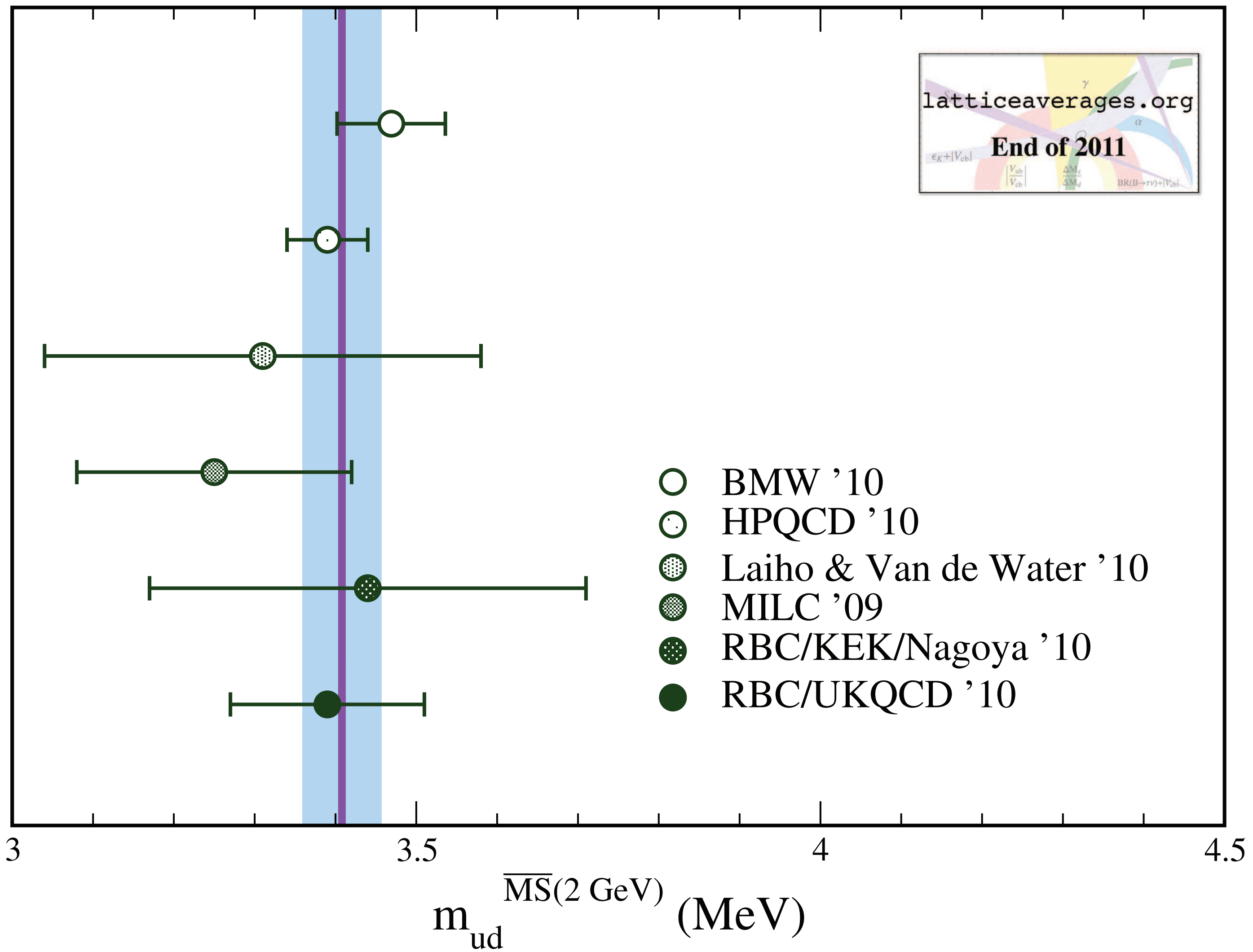
Another class of problems where methods exist, but are more challenging because of disconnected diagrams, more than one meson in the final state, or both. (Maybe 5 years?): $K \to \pi\pi$, ΔM_K, $K \to \pi\ell^+\ell^-$,...

Yet another class of problems where new ideas are needed, and probably a lot more computing: $D \to \pi\pi$, $D \to KK$, $B \to \pi\pi$, D-mixing and CP violation...

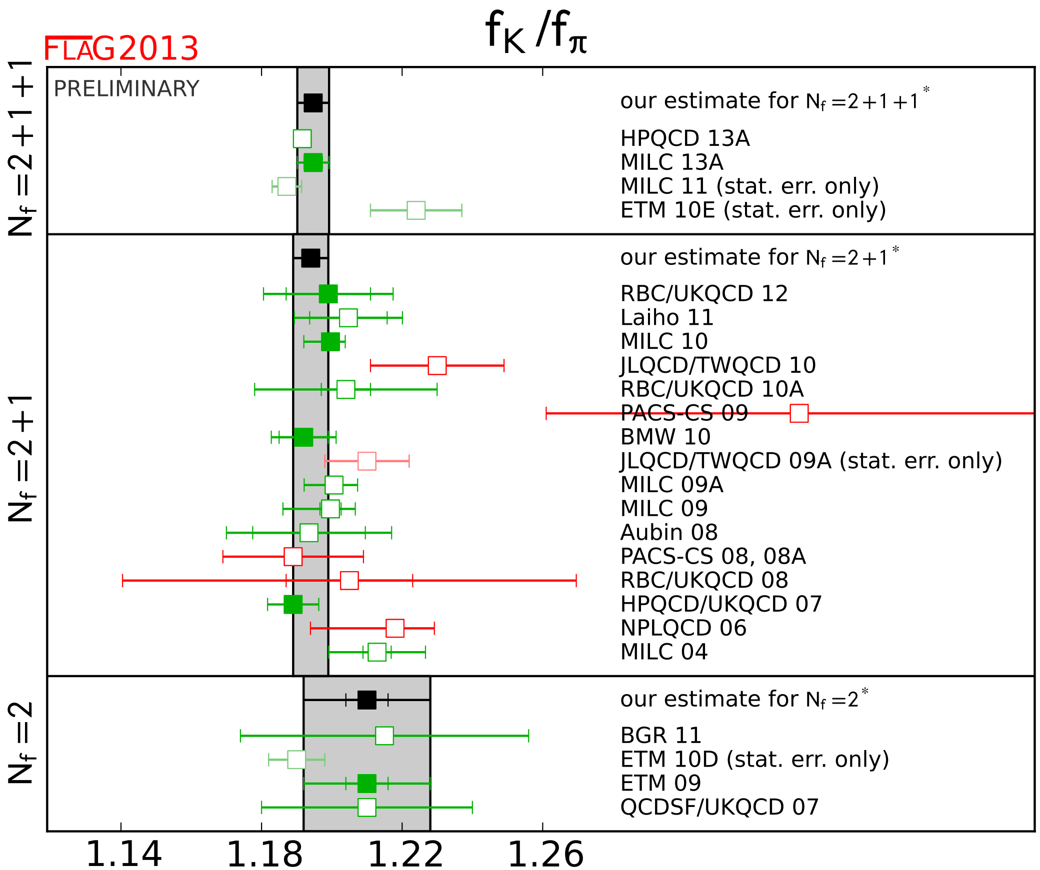
Strange quark mass



Light-quark mass



f_K/f_π



$$\frac{\Gamma(K\to\ell\overline{\nu}_\ell)}{\Gamma(\pi\to\ell\overline{\nu}_\ell)}=\left(\frac{|V_{us}|}{|V_{ud}|}\right)^2\left(\frac{f_K}{f_\pi}\right)^2\frac{m_K\left(1-\frac{m_\ell^2}{m_K^2}\right)^2}{m_\pi\left(1-\frac{m_\ell^2}{m_\pi^2}\right)^2}\left[1+\frac{\alpha}{\pi}(C_K-C_\pi)\right] \tag{5}$$

New results for f_K/f_π

New results from the HISQ (Highly Improved Staggered Quark) action, with lattices generated by the MILC collaboration:

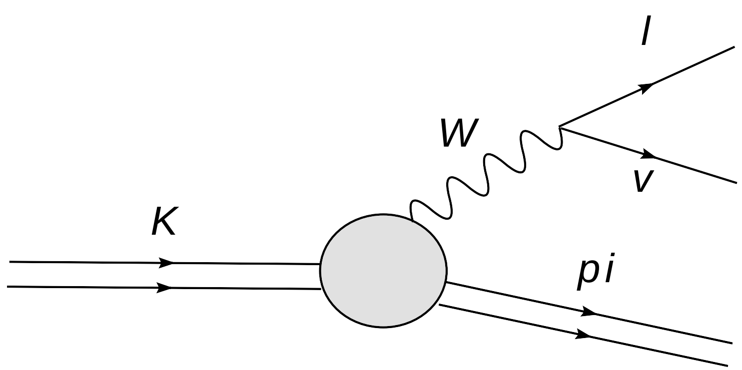
- Physical light quark masses
- Smaller taste-breaking effects than the previous MILC ensembles
- Multiple lattice spacings
- 2+1+1 flavors of sea quarks

MILC: 1.1947(45), PRL 110, 172003 (2013)
HPQCD: 1.1916(21), arXiv:1303.1670

Previous World Average: 1.1936(53)

Errors will soon be dominated by E+M effects. Nontrivial to account for! The lattice theory would include a 4-fermion operator with μ and ν fields explicitly, not just a current between the meson and vacuum.

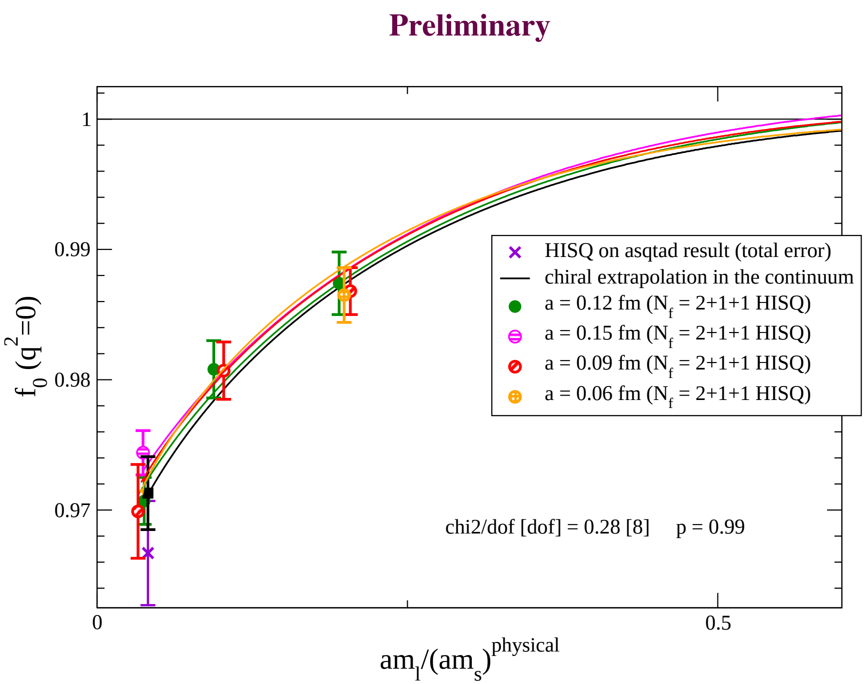
$K \to \pi\ell\nu$

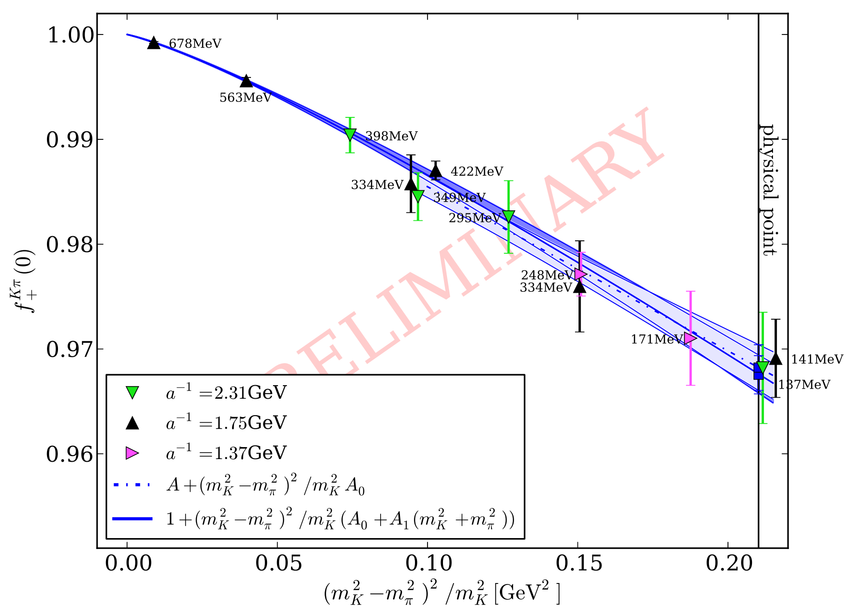


$$\Gamma_{K\ell 3} = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\mathrm{EW}} (|V_{us}| f_+^{K^0\pi^-}(0))^2 I_{K\ell} (1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi})^2, \tag{6}$$

where $S_{EW} = 1.0232(3)$ is the short-distance electroweak correction, C_K is a Clebsch-Gordan coefficient, $f_+^{K^0\pi^-}(0)$ is the form factor at zero momentum transfer, and $I_{K\ell}$ is a phase-space integral that is sensitive to the momentum dependence of the form factors. The quantities $\delta_{\mathrm{EM}}^{K\ell}$ and $\delta_{\mathrm{SU(2)}}^{K\pi}$ are long-distance EM corrections and isospin breaking corrections, respectively.

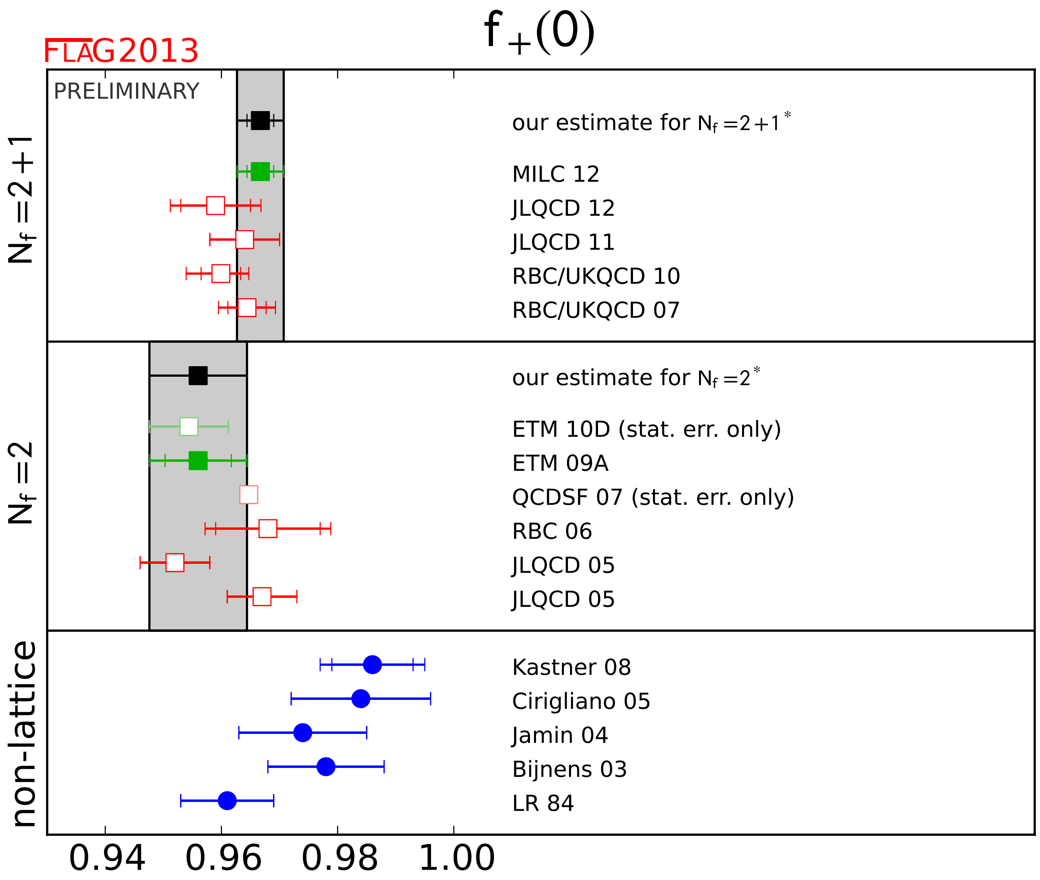
$K \to \pi \ell \nu$





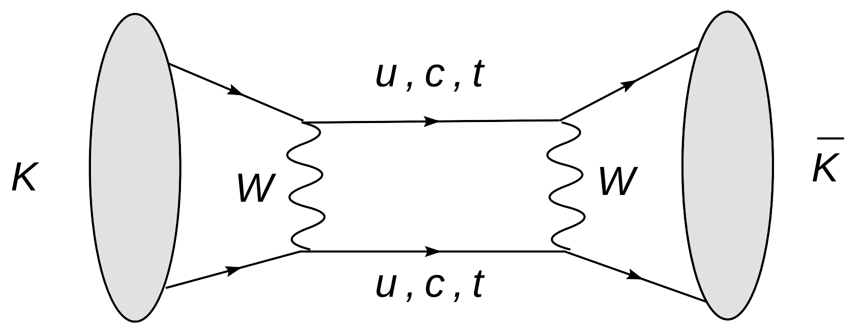
Preliminary results from MILC and RBC/UKQCD

$K \to \pi \ell \nu$



FLAG 2013 preliminary average

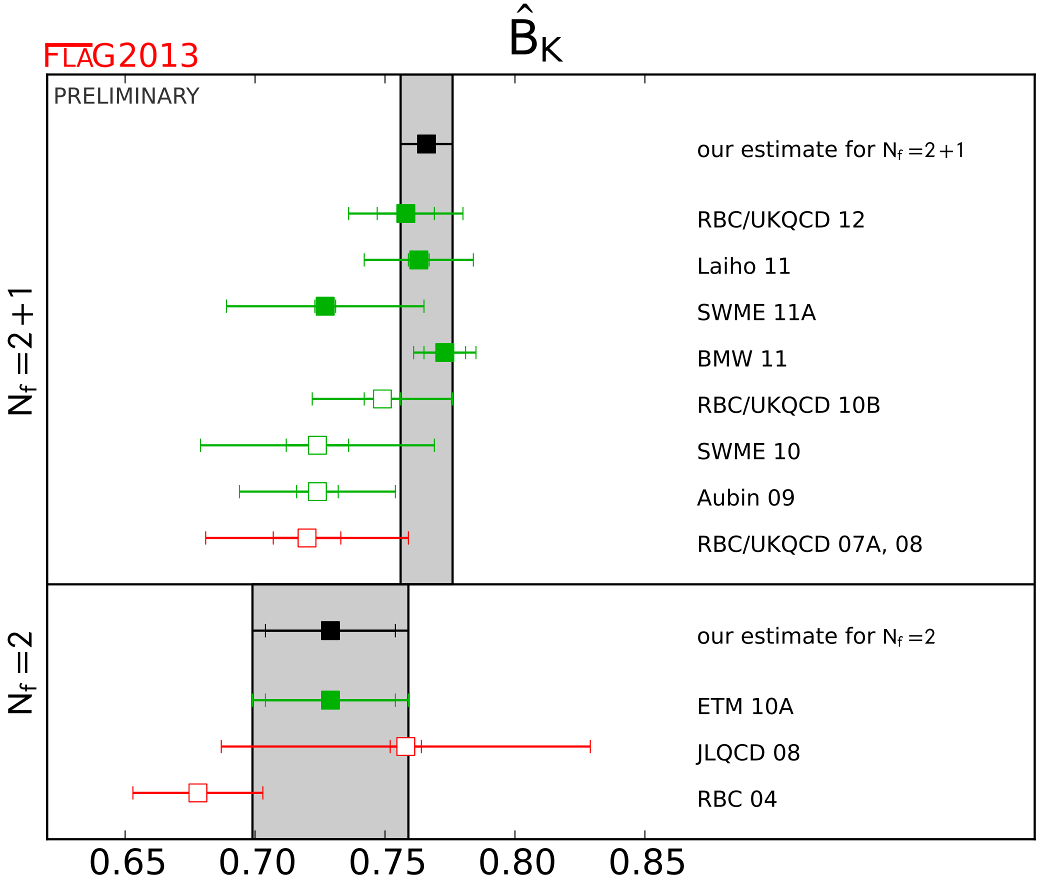
B_K



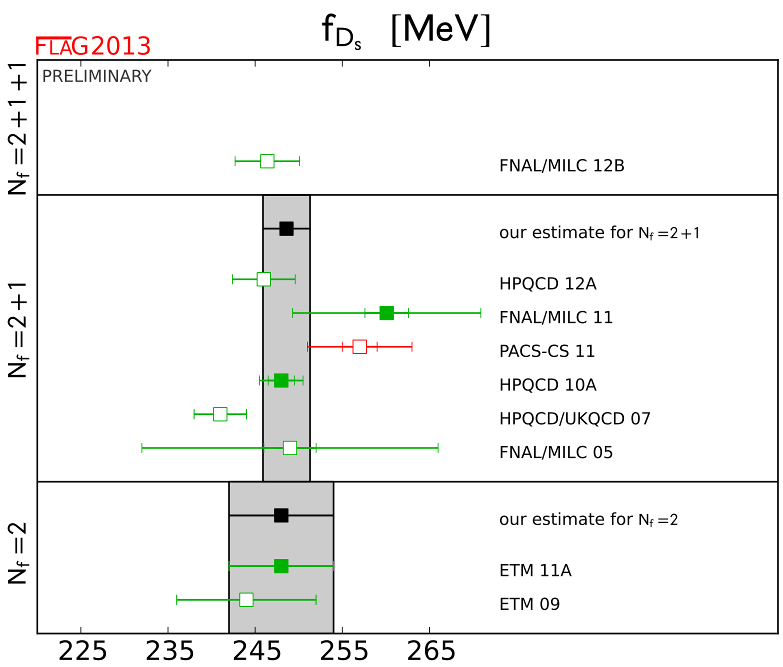
$$|\epsilon_K| = C_\epsilon \kappa_\epsilon B_K A^2 \overline{\eta} \{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

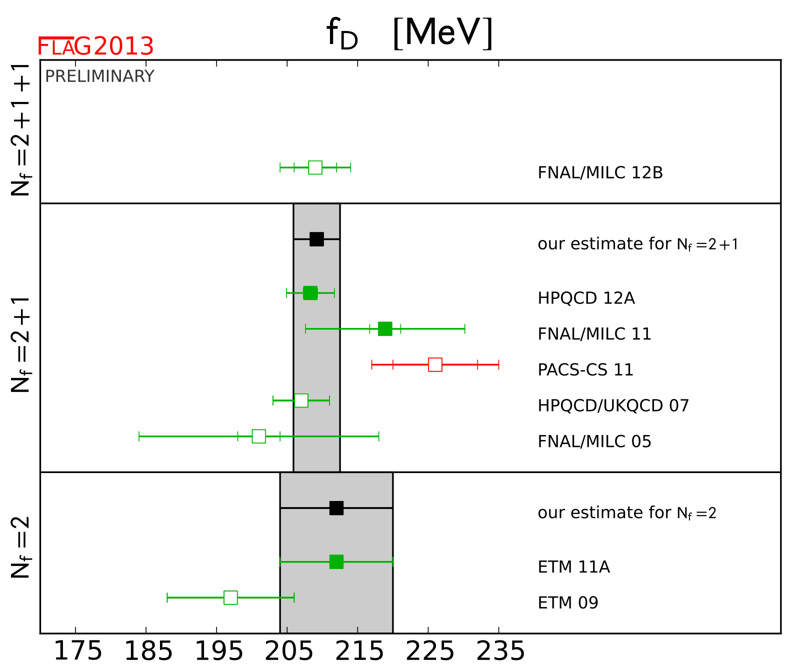
where C_ϵ is a collection of experimentally determined parameters, κ_ϵ represents long-distance corrections and a correction due to the fact that $\phi_\epsilon \neq 45$ degrees, the $\eta_i S_0$ are perturbative coefficients, the terms in blue are CKM matrix elements in Wolfenstein parameterization.

B_K


FLAG2013
$\hat{B}_K$
PRELIMINARY
our estimate for $N_f=2+1$
RBC/UKQCD 12
Laiho 11
SWME 11A
BMW 11
RBC/UKQCD 10B
SWME 10
Aubin 09
RBC/UKQCD 07A, 08
our estimate for $N_f=2$
ETM 10A
JLQCD 08
RBC 04
$N_f=2+1$
$N_f=2$
0.65
0.70
0.75
0.80
0.85

f_{D^+}, f_{D_s}

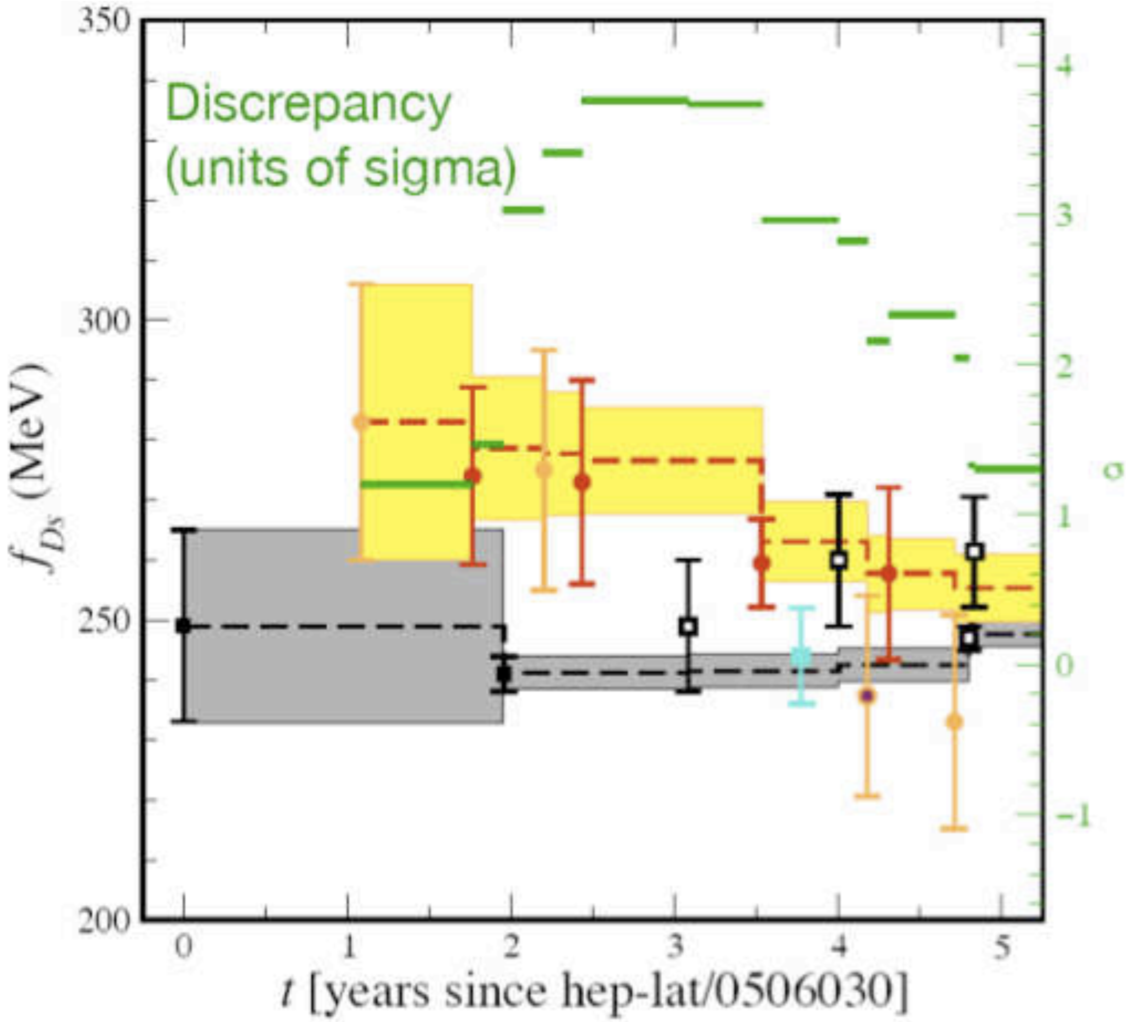




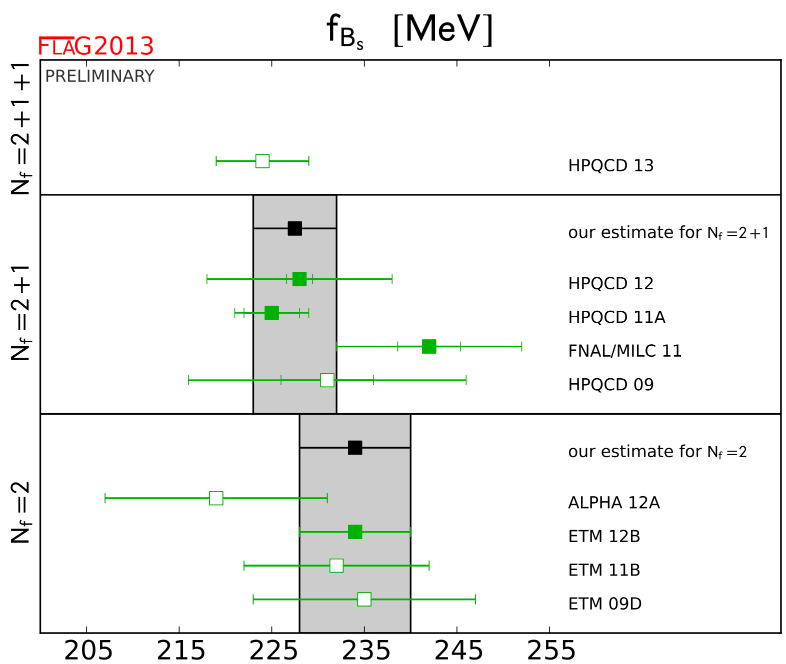
$f_{D_s} = 248.6 \pm 2.7$ MeV, $\quad f_D = 209.2 \pm 3.3$ MeV.

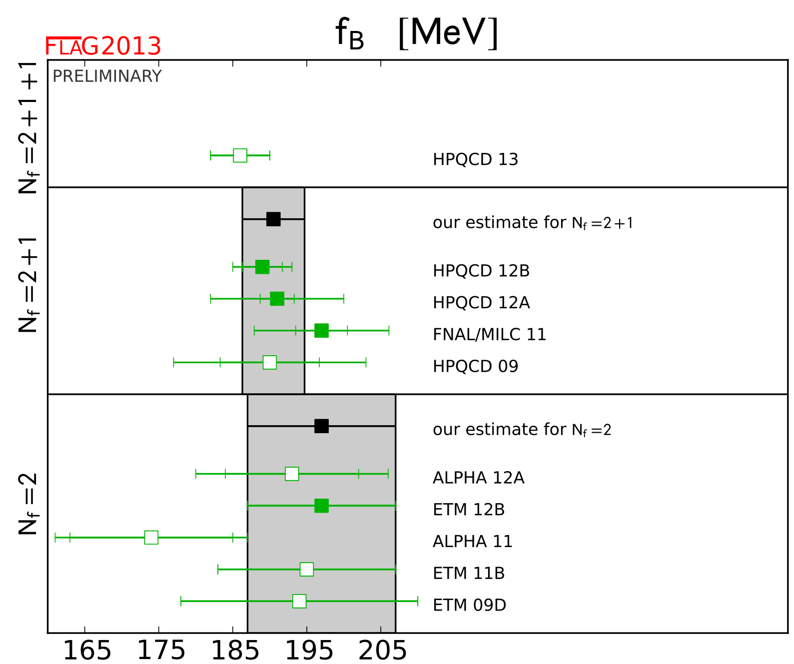
Saga of f_{D_s}

(courtesy of Andreas Kronfeld)



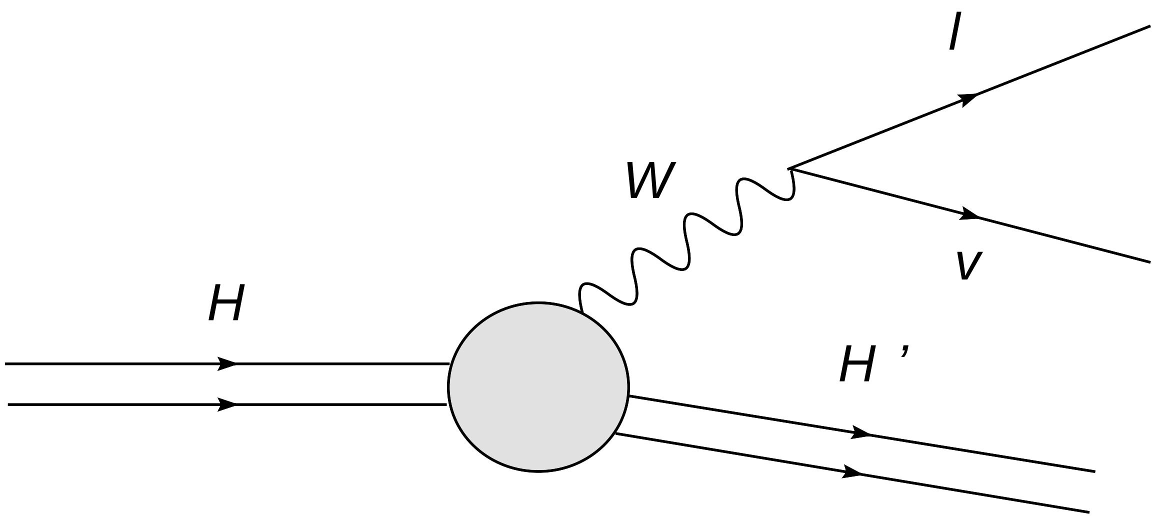
f_B, f_{B_s}





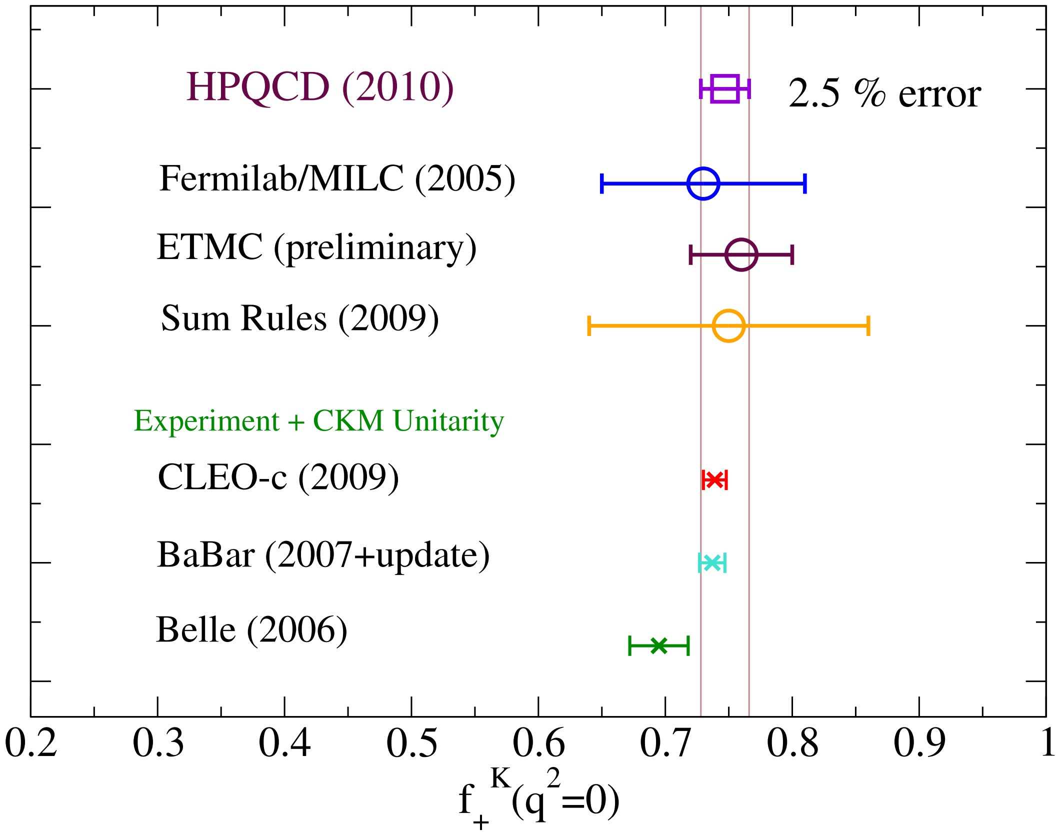
$f_{B_s} = 227.7 \pm 4.5$ MeV, $\quad f_B = 190.5 \pm 4.2$ MeV.

Heavy-light semileptonic decays



Vertex proportional to $|V_{qq'}|$. In order to extract it, a nonperturbative determination of the form factors is needed.

Overview of $D \to K\ell\nu$

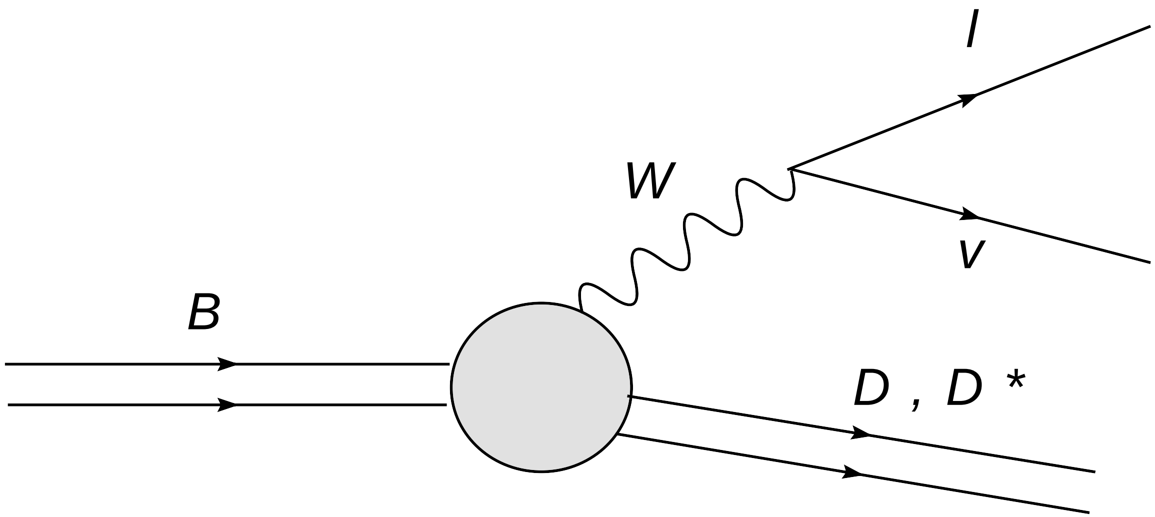


Second row unitarity

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.98(5)$$

$0.234(13) \quad 0.961(26) \quad 39.7(1.0) \times 10^{-3}$

Charmed B semileptonic decays



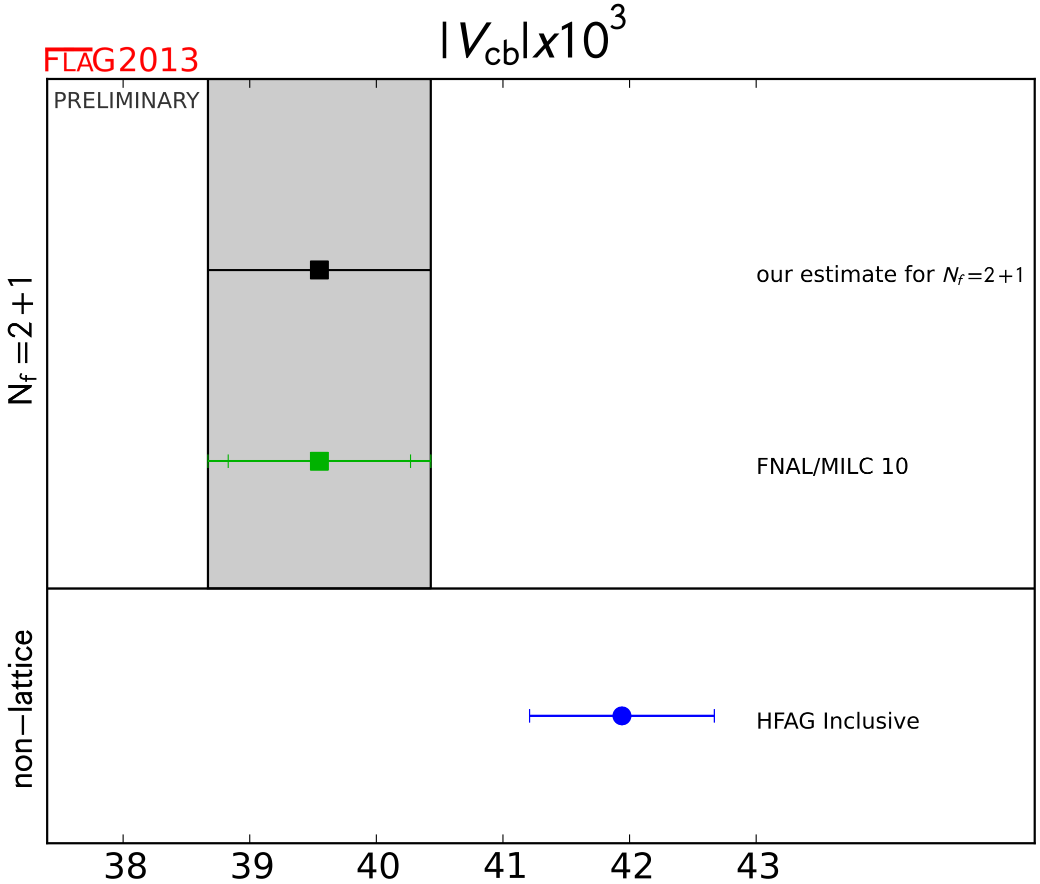
Vertex proportional to $|V_{cb}|$. In order to extract it, nonperturbative input is needed.

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$
\begin{aligned}
\frac{d\Gamma}{dw} &= \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \\
&\quad \times |V_{cb}|^2 \mathcal{G}(w) |\mathcal{F}_{B\to D^*}(w)|^2
\end{aligned}
\tag{7}
$$

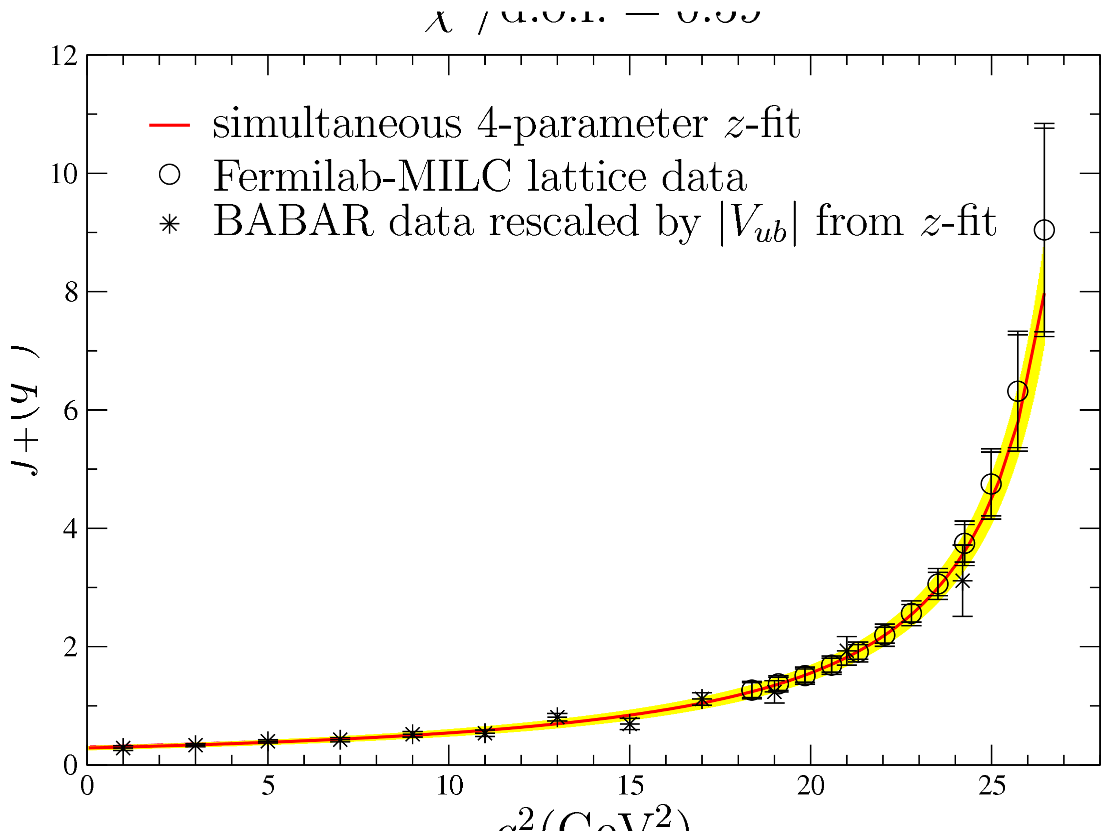
where $\mathcal{G}(w)|\mathcal{F}_{B\to D^*}|^2$ contains a combination of form-factors which must be computed non-perturbatively. $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v').

V_{cb}



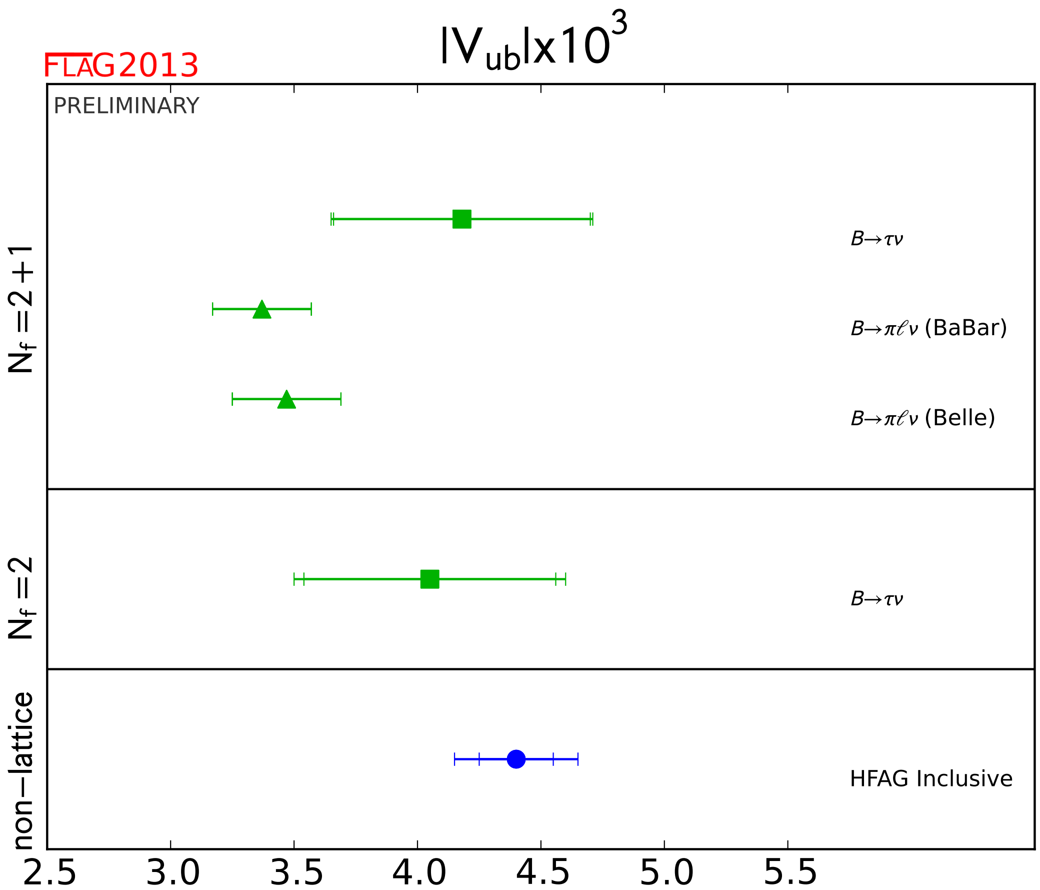
FLAG2013
PRELIMINARY
|Vcb|x10^3
Nf=2+1
non−lattice
our estimate for Nf=2+1
FNAL/MILC 10
HFAG Inclusive
38
39
40
41
42
43

$B \to \pi \ell \nu$



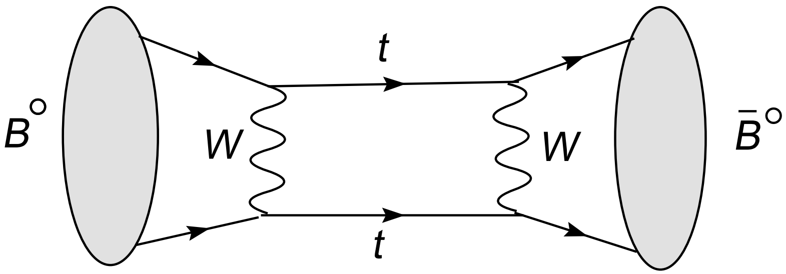
simultaneous 4-parameter z-fit
Fermilab-MILC lattice data
BABAR data rescaled by $|V_{ub}|$ from z-fit
0
2
4
6
8
10
12
0
5
10
15
20
25

V_{ub}



|V_ub|x10^3
FLAG2013
PRELIMINARY
N_f = 2+1
B→τν
B→πℓν (BaBar)
B→πℓν (Belle)
N_f = 2
B→τν
non−lattice
HFAG Inclusive
2.5
3.0
3.5
4.0
4.5
5.0
5.5

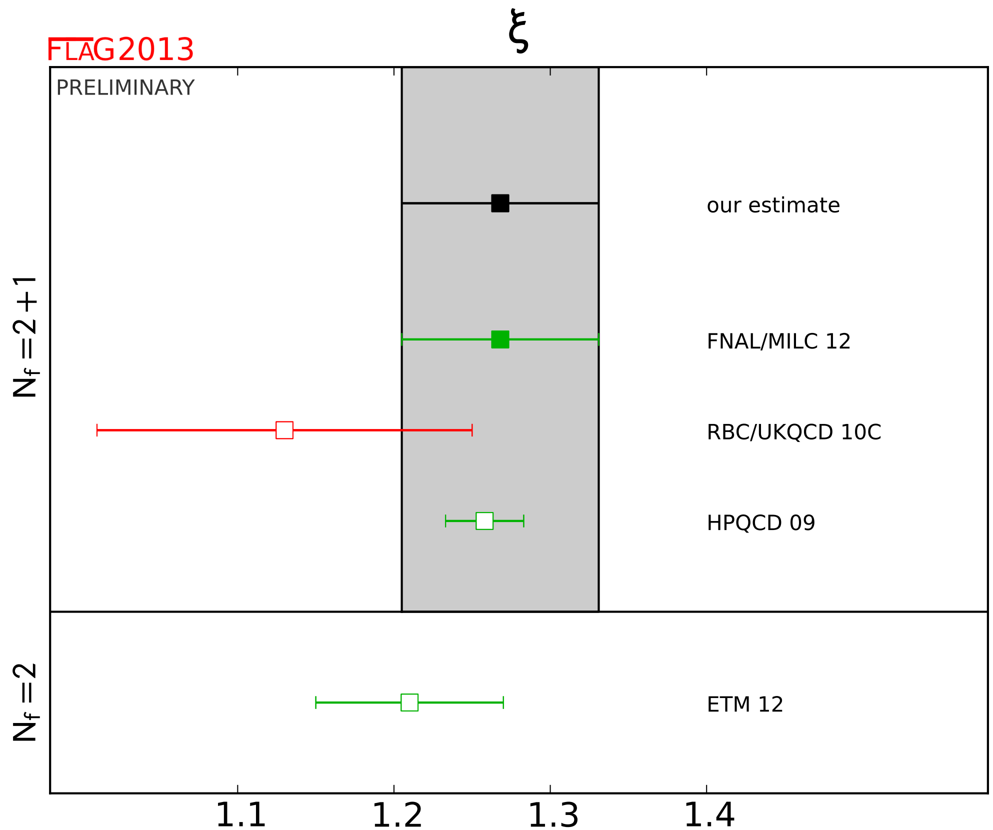
B-$\overline{B}$ Mixing



$$\langle \overline{B}^0 | (\bar{b}d)_{V-A}(\bar{b}d)_{V-A} | B^0 \rangle \equiv \frac{8}{3} m_B^2 f_B^2 B_B, \tag{8}$$

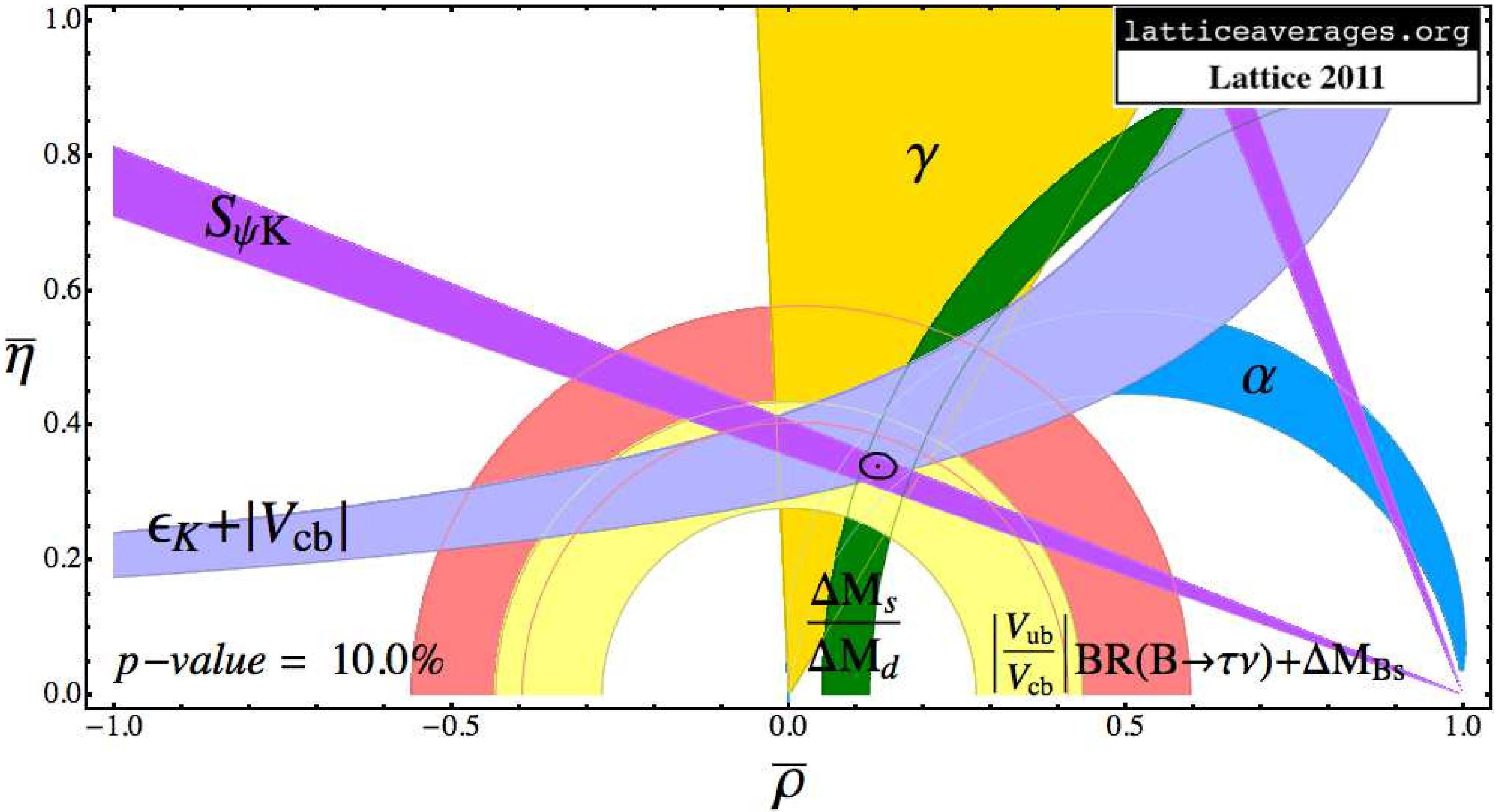
$$\Delta M_s = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \widehat{B}_{B_s} \tag{9}$$

ξ

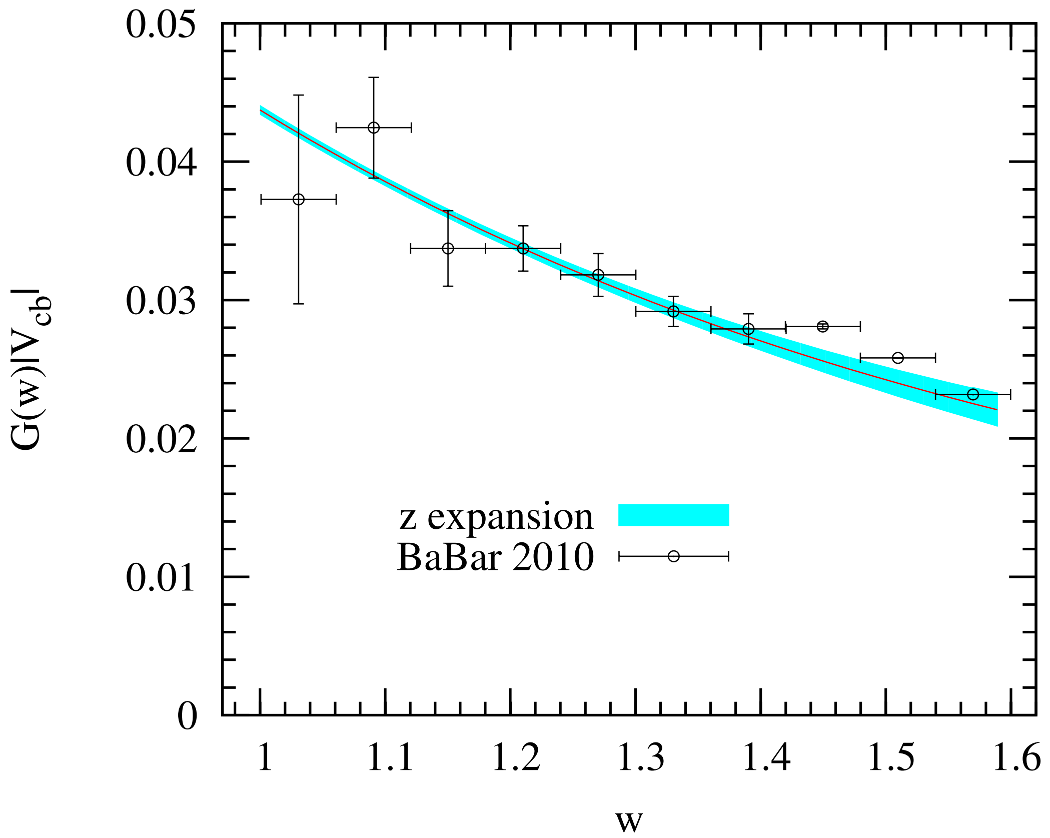


$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.268 \pm 0.063.$$

UT triangle


latticeaverages.org
Lattice 2011
γ
$S_{\psi K}$
α
$\epsilon_K + |V_{cb}|$
$\frac{\Delta M_s}{\Delta M_d}$
$\left|\frac{V_{ub}}{V_{cb}}\right|$ BR(B→τν)+ΔM_{Bs}
p−value = 10.0%
$\bar{\eta}$
$\bar{\rho}$
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0

$B \to D\ell\nu$ at non-zero recoil



A comparison of the form factor shape using lattice calculations and the z expansion to BaBar data assuming $|V_{cb}| = 41.4 \times 10^{-3}$. More data and analysis in progress for precision determination of $|V_{cb}|$.

$R(D)$

BaBar has measured:

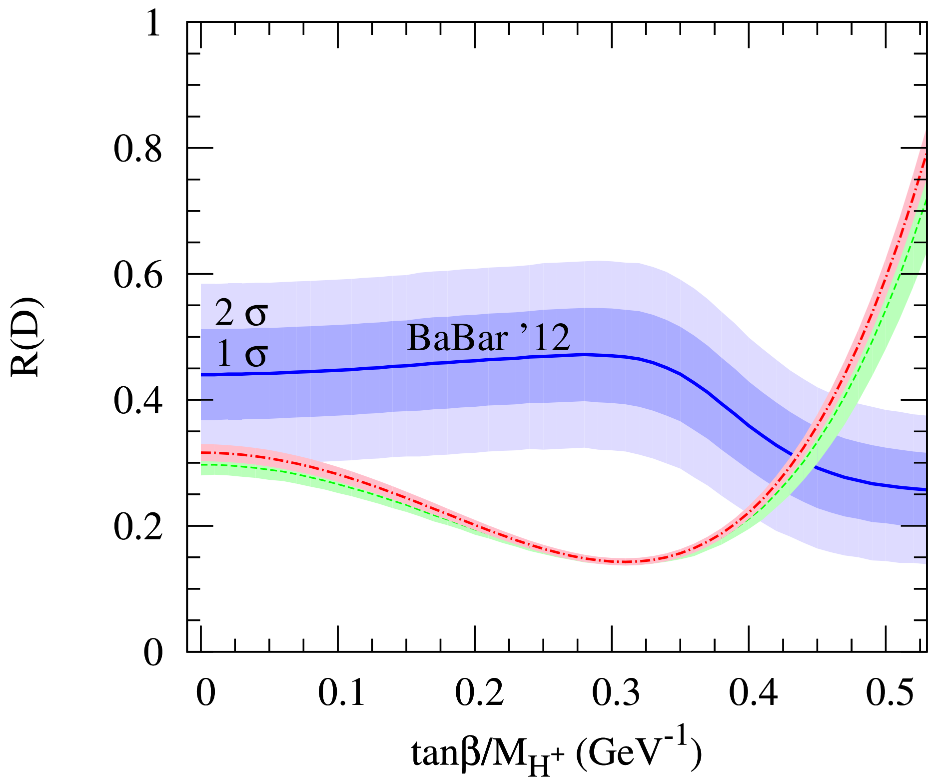
$$R(D^*) = \frac{\mathcal{B}(B \to D^* \tau \nu)}{\mathcal{B}(B \to D^* \ell \nu)} = 0.332 \pm 0.030, \tag{10}$$

$$R(D) = \frac{\mathcal{B}(B \to D \tau \nu)}{\mathcal{B}(B \to D \ell \nu)} = 0.440 \pm 0.072. \tag{11}$$

Standard Model predictions are $R(D^*) = 0.252(3)$ and $R(D) = 0.296(16)$ [Fajfer, et. al., arXiv:1203.2654] using kinematic and dispersive constraints on the shape and HQET to relate the unmeasured f_0 to the measured f_+. (Note $|V_{cb}|$ cancels in the ratio in the SM.)

Can do (first) unquenched lattice calculation of $R(D)$ as spin-off of previous work.

$R(D)$



Prospects for the future

Error forcast

Quantity	CKM element	Present expt. error	Present lattice error	2014 lattice error	2018 lattice error
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.5%	0.3%	0.15%
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	0.5%	0.35%	0.2%
f_D	$\lvert V_{cd}\rvert$	4.3%	2%	1%	$< 1\%$
f_{D_s}	$\lvert V_{cs}\rvert$	2.1%	2%	1%	$< 1\%$
$D \to \pi\ell\nu$	$\lvert V_{cd}\rvert$	2.6%	4.4%	3%	2%
$D \to K\ell\nu$	$\lvert V_{cs}\rvert$	1.1%	2.5%	2%	1%
$B \to D^*\ell\nu$	$\lvert V_{cb}\rvert$	1.3%	1.8%	1.5%	$< 1\%$
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	8.7%	4%	2%
f_B	$\lvert V_{ub}\rvert$	9%	2.5%	1.5%	$< 1\%$
ξ	$\lvert V_{ts}\rvert/\lvert V_{td}\rvert$	0.4%	2.5%	1.5%	$< 1\%$
ΔM_s	$\lvert V_{ts}V_{tb}\rvert^2$	0.24%	11%	8%	5%
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	1.3%	1%	$< 1\%$

Conclusions

Simple quantities from many different groups using different methods can be calculated with controlled systematic errors.

Many quantities are now precision calculations, and results are in good agreement. Prospects for improvement are excellent!

More difficult quantities like $K \to \pi\pi$ and ΔM_K are becoming possible (as Soni will discuss). Many new things (including long-distance effects in various kaon decays) to do!

Backup Slides

Some Ongoing Lattice Projects

Group	N_f	action	a(fm)	$m_\pi L$	$m_\pi^{\rm min}$ (MeV) sea/val
ETMC	2(+1+1)	Twisted Mass	0.05-0.10 fm	$\gg 1$	$280/280$
MILC	2+1(+1)	staggered	0.045-0.12 fm	> 4	$130/140$
RBC/UKQCD	2+1	Domain Wall	0.085-0.15 fm	> 4	$180/135$
JLQCD	2+1	Overlap	0.11 fm	≥ 2.7	$310/310$
PACS-CS	2+1	Clover	0.09 fm	≥ 2.0	$140/140$
BMW	2+1	Clover	0.054-0.125 fm	≥ 4	$135/135$
HPQCD	2+1(+1)	staggered	0.045-0.15 fm	> 4	$130/170$

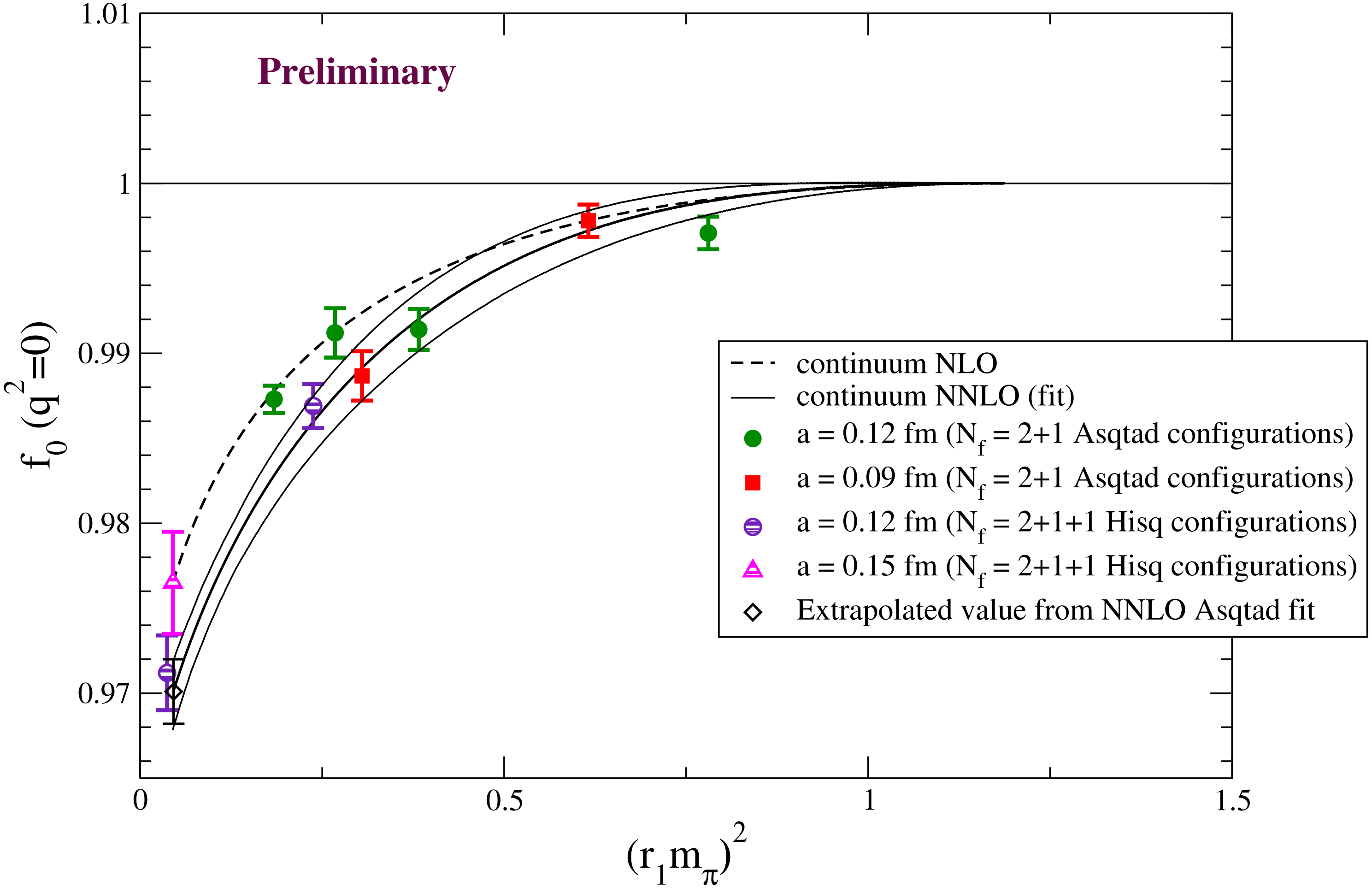
(In staggered calculations, the sea pion mass quoted is the rms value. The valence pion mass quoted is the taste-goldstone.)

Criteria for FLAG II

- Only quantities that are documented in publications are included.
- Only quantities that include complete statistical and systematic error budgets are included in the averages.
- Averages only include 3 (or 4) flavor numbers. It is difficult to assess the error due to quenching the strange quark, and the $\sim$percent level precision for some averages is approaching the size that one would expect for this effect.

Coming soon!

MILC/Fermilab $f_+(0)$



B_K calculations

- RBC/UKQCD: Domain wall fermions, new non-perturbative renormalization scheme (non-exceptional momenta) with smaller systematic errors. 3 lattice spacings (1 with different action).
- JL and Van de Water: Mixed action domain wall on staggered. Adopted new renormalization scheme of RBC/UKQCD. 3 lattice spacings.
- SBW: improved staggered action on the MILC (staggered) ensembles and 4 lattice spacings. Perturbative matching.
- BMW: Clover fermion action, so explicit chiral symmetry breaking. Wrong-chirality operators are small (1%) due to smearing of the action. Non-perturbative matching to perturbation theory performed at high scale to minimize matching error. Physical light quark masses and large volumes. 4 lattice spacings.

$K \to \pi\pi$

$$\text{Re}(\varepsilon'_K/\varepsilon) \sim \frac{\omega}{\sqrt{2}|\varepsilon_K|}\left(\frac{\text{Im}(A_2)}{\text{Re}(A_2)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)}\right) \tag{12}$$

where $\omega = \text{Re}(A_2)/\text{Re}(A_0)$, $A(K^0 \to \pi\pi(I)) = A_I e^{i\delta_I}$.

$\text{Re}(A_2)$ is well known experimentally, and serves as a benchmark.

ε'/ε requires the $\Delta I = 1/2$ channel. This could provide a very important constraint on new physics with reduced hadronic uncertainties. How well does lattice have to do in the $\Delta I = 1/2$ channel to be interesting to phenomenology?

$$\text{Re}(\varepsilon'_K/\epsilon_K) = (1.68 \pm 0.19) \times 10^{-3} \tag{13}$$

Experiment is known to $\sim 10\%$. Even a 30% theory error would be interesting.

$K \to \pi\pi$ matrix elements on the lattice

Maiani-Testa no-go theorem tells us that we cannot extract physical matrix elements from Euclidean correlation functions with multi-hadron states.

Difficulties simulating at physical kinematics for $K \to \pi\pi$ matrix elements avoided by using Lellouch-Lüscher finite volume method. This is still costly. Most straightforward implementation requires a large (6 fm) box, momentum insertion, and physical light quark masses.

RBC/UKQCD

Direct approach of Lellouch-Lüscher.

Calculation on $32^3 \times 64 \times 32$ (DSDR) domain wall fermion ensembles, with $a^{-1} = 1.4$ GeV and 4.5 fm box.

To give the pions momentum without having to fit excited states, twisted boundary conditions are used (Kim and Christ, Lattice 2002 [hep-lat/0210003], Sachrajda and Villadoro hep-lat/0411033).

RBC/UKQCD Re(A_2) and Im(A_2)

uncertainty	Re(A_2)	Im(A_2)
statistics	4.3%	7.5%
finite lattice spacing	15%	15%
finite volume errors	6.2%	6.8%
Partial quenching effect	3.5%	1.7%
operator renormalization	1.7%	4.7%
unphysical kinematics	3.0%	0.22%
derivative of the phase shift	0.32%	0.32%
Wilson coefficient	7.1%	8.1%
total	19%	20%

Results presented in PRL 108 (2012) 141601.

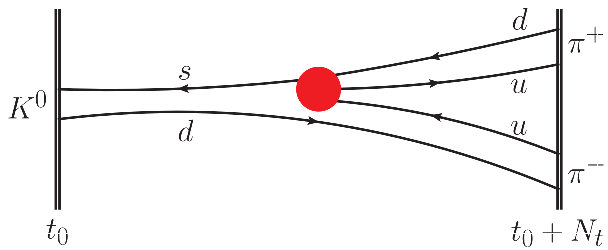
Complications of $\Delta I = 1/2$ channel

1) Power divergences. These can be handled by a vacuum subtraction, as shown by RBC Collaboration in the quenched approximation. Important to have chiral (expensive) quark discretization.

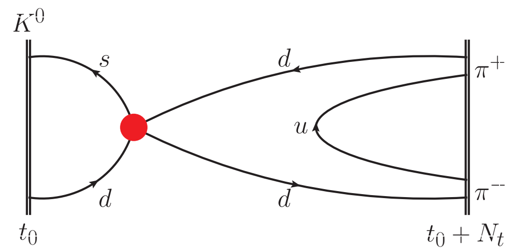
2) Enhanced finite volume effects. Can be controlled by using the unitary points (i.e. no quenching or partial quenching).

3) Disconnected graph. Requires brute force computing. Contributes at NLO in the $SU(3)$ chiral expansion, so nominally sub-leading.

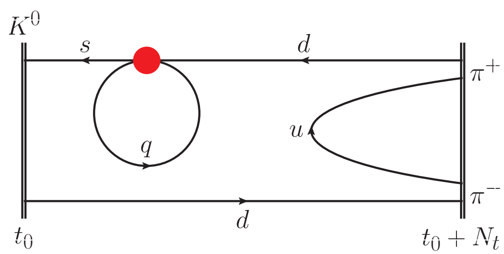
Diagrams for $\Delta I = 1/2$ channel



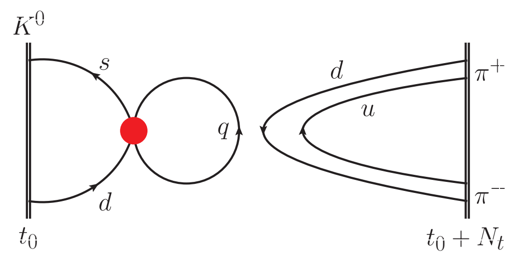
(a)



(b)



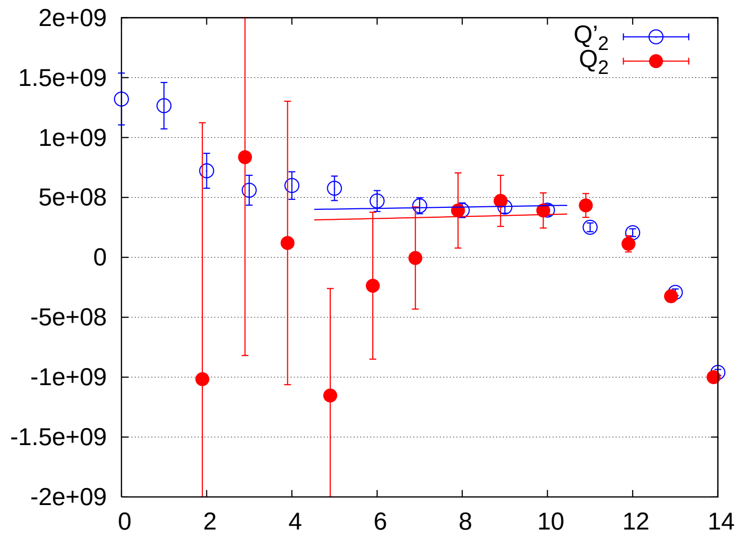
(c)

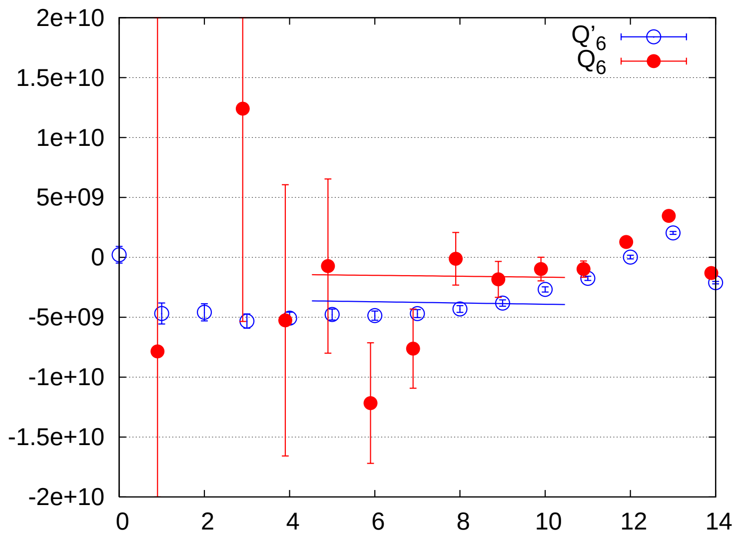


(d)

Signals for $K \to \pi\pi$

Results from RBC/UKQCD, 2010

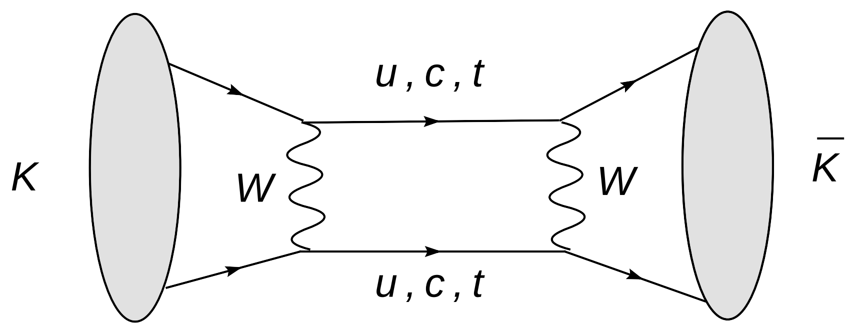




Signals for $K \to \pi\pi$ matrix elements at zero momentum for Q_2 [relevant for Re(A_0)] and Q_6 [relevant for Im(A_0)]. Filled symbols include disconnected diagrams and open symbols do not. Propagators were inverted on each time slice ($T = 32$) for 400 configurations.

Improvements have been made [RBC/UKQCD, PRL 110 152001 (2013)], and a signal can now be resolved for Im(A_0) . Still at unphysical kinematics, but additional improvements and a new, bigger machine will help.

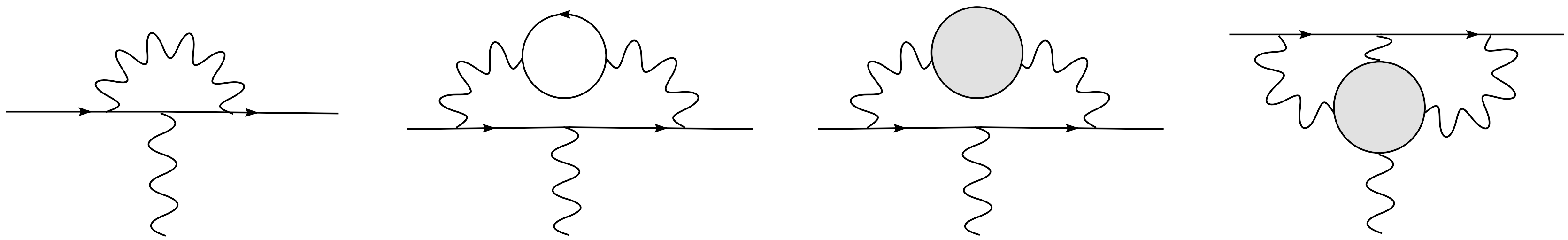
ΔM_K



If W's are contracted to points leaving the internal u, c quark lines soft, this leads to long-distance corrections. Small (few $\%$) in ϵ_K but larger (10-30$\%$?) in ΔM_K. Requires double insertion of 4-quark operators on the lattice. Method and prototype calculation given by RBC/UKQCD in arXiv:1212.5931.

- Extension of Lellouch-Luscher method to second order in weak interactions.
- Proof of principle extraction of physical matrix elements from lattice correlation functions.
- Disconnected diagrams omitted, but could be included with enough computing.

Muon $g-2$



a b c d

a + b) QED known to 4 loops, EW known to 2 loops.

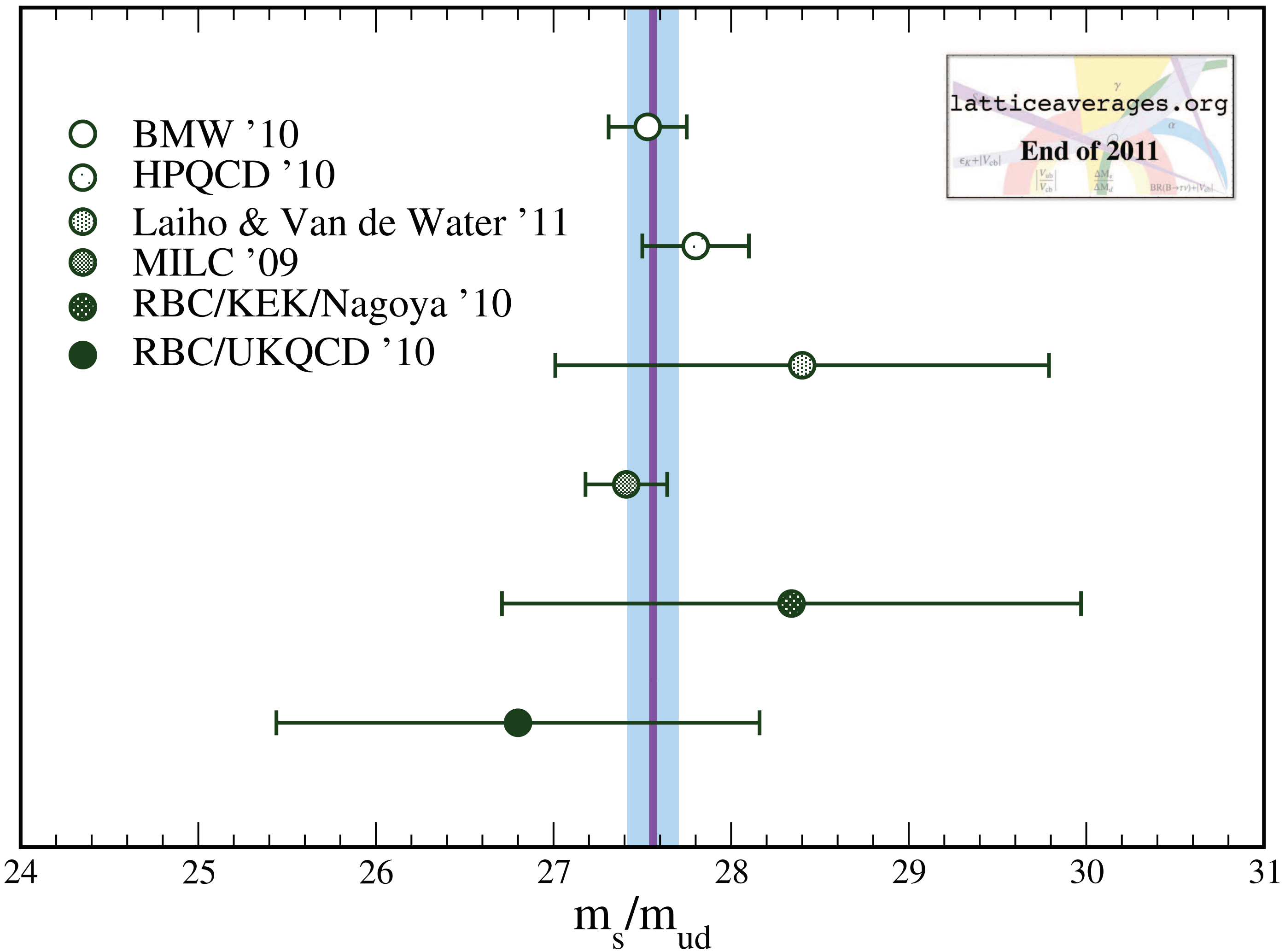
c) Hadronic vacuum polarization (HVP) known from experimental result for $e^+e^- \to$ hadrons plus dispersion relation

d) Hadronic light-by-light (HLbL) estimated from models such as large N_c, vector meson dominance, etc...

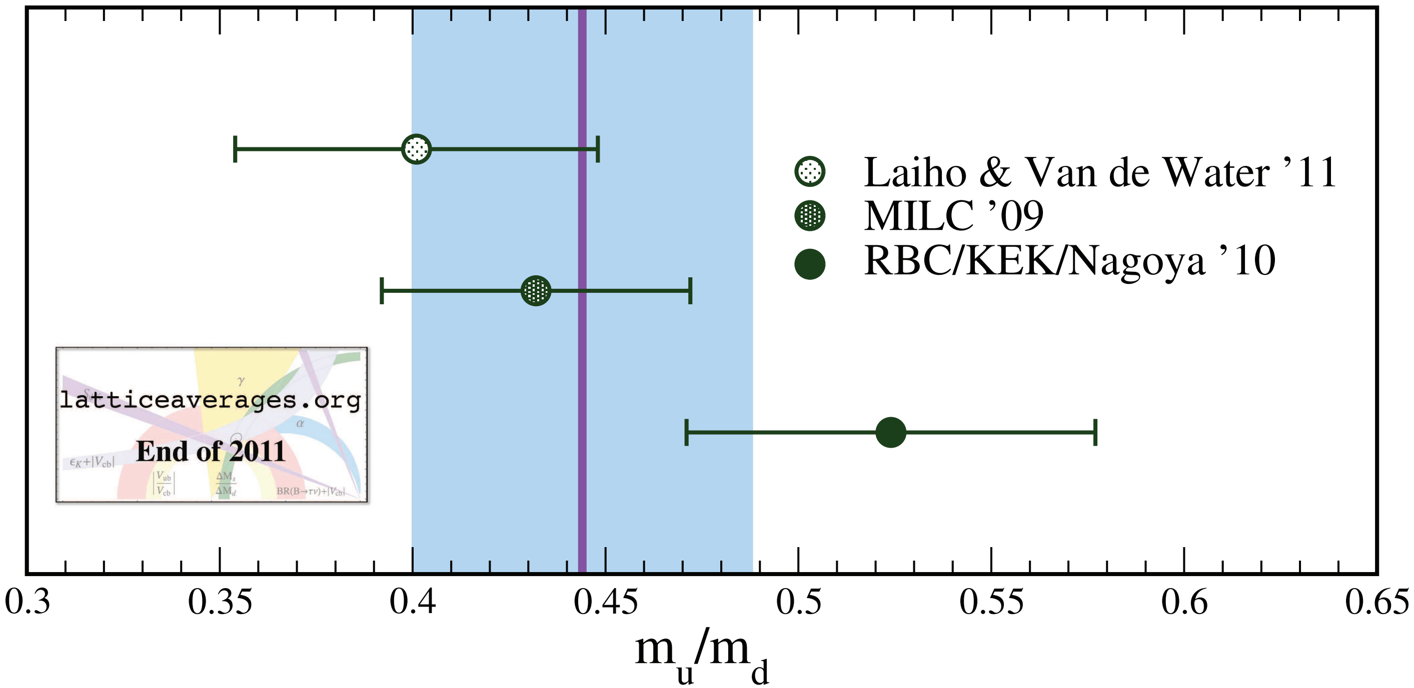
Errors in HVP and HLbL dominate the muon $g-2$ error. Improvements in planned Fermilab $g-2$ will require improved precision, and lattice can help here. Will need 0.2% on HVP and $10-15\%$ on HLbL.

More computing, and perhaps new ideas needed for HLbL.

Quark mass ratio



Light quark mass ratio



Errors inflated by ~ 1.4 due to somewhat low confidence level. Still $\sim 10\sigma$ from zero.

$K \to \pi\pi$, $\Delta I = 3/2$, $(27, 1)$